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"SAROS" AND LUNAR VELOCITY IN BABYLONIAN ASTRONOMY

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Synopsis.

Babylonian ephemerides for the moon, written in the Hellenistic period, contain a column which is periodic with the period of the lunar anomaly. On the basis of a text recently discovered by A. Sachs in the British Museum, it is now possible to show that essential parameters for the determination of this periodic function are connected with the so-called Saros cycle of 223 mean synodic months.

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Introduction.

We have today a fair knowledge of the procedures by means of which the Babylonian astronomers of the last three centuries B.C. determined the moments of the syzygies.[1] Almost nothing, however, is known about the empirical or theoretical steps which led to the final theory. Only a few texts are preserved[2] which concern the earlier phases of mathematical astronomy, and their terminological and other difficulties are so great that we are still far from a real understanding of their contents. Nevertheless, this much is obvious: the 18-year eclipse cycle, commonly called "Saros",[3] plays an important role in this earlier development but is no longer apparent in the final form of the theory.

Our comparatively detailed insight into the latest phase of Babylonian lunar theory is marred by a particularly puzzling problem. The procedures of "System A"—considered to be the earlier of the two lunar systems (with how much justification, I do not know)—employ a column, which I denote by Φ, for the computation of the length of the synodic month. Until now we have been unable to explain the significance of its parameters except for the fact that its period is the same as the period of column F of the lunar velocity[4]. This is particularly disturbing since not only do we have a great number of texts dealing with the relations between Φ, F, and G[5]—the latter giving the excess of the synodic month over 29 days—but there exist additional tables of a closely related function Λ of equally unknown significance[6].

It is therefore of great interest that we now have a text which connects the elements of column Φ with the 18-year cycle. This fact was recognized

[1] All material on Babylonian mathematical astronomy known to me so far is collected in my edition "Astronomical Cuneiform Texts" (3 vols., London, Lund Humphries, 1955), henceforth quoted as ACT.

[2] Example: TU 11 = AO 6455 (Thureau-Dangin, Tablettes d'Uruk, Paris 1922, Pl. 22 f.).

[3] This name is a modern invention; cf., e. g., my "Exact Sciences in Antiquity", Copenhagen 1951, p. 134 ff.

[4] For details cf. ACT I p. 44 and p. 58 ff.

[5] Cf. ACT II p. 508 s.v. column Φ.

[6] Cf. ACT I p. 264 ff.

by A. SACHS during his investigation of astronomical texts in the British Museum in 1954. He joined two fragments BM 36705 = 80-6-17,437 and BM 36725 = 80-6-17,458 which give us, except for broken edges and some surface damage, the major part of a text which contained in some 16 sections the theory of column Φ. Needless to say, the damage to the critical passages of the text is such that we have failed in repeated discussions to reach a consistent and thoroughgoing understanding. Nevertheless the mere fact that we now see a connection between the Saros and column Φ explains certain features in the lunar theory and opens a new approach which should be made accessible to others who might be more fortunate in finding the key to the final solution of the problem of column Φ.

The crucial passage which establishes the relation between Saros and lunar velocity is the following sentence in our text (rev. 13 and 16): 17,46,40 tab *u* lal *šá* 18 mu-meš "17,46,40 is the difference (lit.: addition and subtraction) for 18 years". It had been apparent from many texts that the constant

$$\varphi = 17{,}46{,}40{,}0^{7} \tag{1a}$$

plays an important role in the structure of column Φ; e.g., we knew[8] that the difference d_Φ of Φ was given by

$$d_\Phi = \frac{1}{\varepsilon}\varphi \tag{1b}$$

where

$$\varepsilon = \frac{1}{9;20} = \frac{3}{28} \tag{1c}$$

is a constant which repeatedly appears in related contexts. Similar connections exist between φ and the function Λ. But the sentence quoted above reveals a new relation which had been overlooked and which now can be formulated as follows: If the difference $y_n - y_o$ between the two values y_n and y_o of Φ is $\varphi = 17{,}46{,}40{,}0$, then we can determine the number n of mean synodic months which separate y_n from y_o and the corresponding number α of periods of length P_Φ of Φ. Indeed, using the standard Diophantine method,[9] we find

$$n \equiv -3{,}43 \text{ mod. } 1{,}44{,}7 \qquad \alpha \equiv -16 \text{ mod. } 7{,}28. \tag{2}$$

[7] The final zero is added in order to indicate that the digits of φ correspond to the last four digits of Φ, taken as integers. As we shall see presently $\varphi = 0;0{,}17{,}46{,}40^{\text{H}}$ where $1^{\text{H}} = 1{,}0^\circ$ of time (cf. ACT I p. 39).

[8] Cf. ACT I p. 59.

[9] Cf. ACT I p. 35 f.

This is identical with the statement of our text since

$$S = 3{,}43 \text{ mean synodic months} \tag{3}$$

is the length of the "Saros".

Utilizing this new insight into the significance of the constant φ we can derive several important relations. First of all we can show that (1b) and (1c) are the equivalent of the fundamental period relation between anomal-

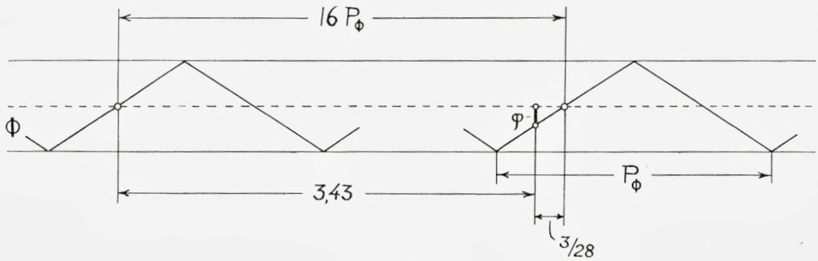


Fig. 1.

istic and synodic months. Indeed, (1b) implies that the increase of Φ by the amount φ corresponds to the fraction $\varepsilon = \dfrac{3}{28}$ of one synodic month (cf. Fig. 1). Thus (1) and (2) combined indicate that

$$16\, P_\Phi = 3{,}43 + \frac{3}{28} \text{ syn. m.} \tag{4}$$

or

$$16 \cdot 28\, P_\Phi = 3{,}43 \cdot 28 + 3 = 1{,}44{,}7 \text{ syn. m.,}$$

thus

$$P_\Phi = \frac{1{,}44{,}7}{7{,}28}. \tag{5}$$

This is indeed the value known from the ephemerides of System A. Since F and Φ agree in period and in phase[10] we obtain from (5) for the anomalistic months the well-known relation[11]

$$1{,}44{,}7 \text{ syn. m.} = 1{,}44{,}7 + 7{,}28 = 1{,}51{,}35 \text{ anom. m.} \tag{6}$$

Consequently the relation (6) is the exact equivalent of the statements (1b) and (1c).

[10] Cf. ACT I p. 44 and p. 58.
[11] Cf. ACT I p. 58 (1c).

The relation

$$(7)\qquad 3{,}43 \text{ syn. m.} = 3{,}59 \text{ anom. m.}$$

which is usually caled "Saros" is only an approximation of (6) since division by 28 of (6) leads to

$$(8)\qquad 3{,}43 + \frac{3}{28} \text{ syn. m.} = 3{,}59 + \frac{3}{28} \text{ anom. m.}$$

and not to (7), which ignores the difference in length of the fractions of synodic and anomalistic months. In the following we shall use the word "Saros" exclusively as an abbreviation of (3), that is, for a time interval of 3,43 synodic months or about 18 years, as the texts usually say. We do not imply, however, any exact equivalence of this time interval with any other integer number of units.

Another important relation can be derived from the fact that Φ and F have identical periods and therefore so do also Φ and $\hat{\mathrm{G}}$ since the identity of the periods of F (lunar velocity) and $\hat{\mathrm{G}}$ (linearized length of the synodic months assuming mean solar motion) is astronomically necessary. From the identity of periods we can now explain the fact[12] that the texts assume

$$(9)\qquad d_{\hat{\mathrm{G}}} = \frac{1}{\varepsilon}\, d_{\Phi}.$$

Indeed, for the common period we have from (5):

$$P = \frac{2\,\Delta}{d} = \frac{\Pi}{Z} = \frac{1{,}44{,}7}{7{,}28}$$

and with (3)

$$S = 3{,}43$$

one finds

$$\Pi = 28\,S + 3 \qquad Z = 28 \cdot 16.$$

We now define the difference d_{Φ} of Φ by

$$(10)\qquad -\,d_{\Phi} = \hat{\mathrm{G}}_S - \hat{\mathrm{G}}_0.$$

Since for the linear zigzag function $\hat{\mathrm{G}}$

$$\hat{\mathrm{G}}_S - \hat{\mathrm{G}}_0 \equiv S \cdot d_{\hat{\mathrm{G}}} \qquad \mathrm{mod}\ 2\,\Delta_{\hat{\mathrm{G}}}$$

and since

$$S = \frac{\Pi - 3}{28} = \frac{16\,\Pi}{Z} - \frac{3}{28} = 16\,\frac{2\,\Delta}{d} - \varepsilon$$

[12] Cf. ACT I p. 59 (4) and the table ACT I p. 60. The steps of length ε are recognizable in this table by the corresponding change of Φ by the amount 17,46,40,0.

we have from (10)

$$- d_\Phi \equiv S\, d_{\hat{G}} = 16 \cdot 2\, \Delta_{\hat{G}} - \varepsilon\, d_{\hat{G}} \equiv - \varepsilon\, d_{\hat{G}} \qquad \mathrm{mod}\ 2\, \Delta_{\hat{G}}$$

or

$$d_\Phi = \varepsilon\, d_{\hat{G}}$$

q.e.d. Thus we have shown that the difference of Φ is the difference in the length of two synodic months one Saros apart.

A trivial consequence of the preceding derivation is the fact that not only is d_Φ derived from the variation of the length of the synodic months but Δ_Φ as well. Indeed, in every linear zigzag function,

$$\Delta = \frac{1}{2} P d$$

and since $P_\Phi = P_{\hat{G}}$ we know also Δ_Φ since d_Φ is now known.

In order to determine Φ completely, two more data would be required: its mean value and some initial value, or its equivalent, the phase difference between Φ and G. I have not succeeded in solving these problems. There are several sections in our new texts, particularly 12 and 13, which deal with the amplitudes of Ĝ and Φ and what seem to be maxima and minima which would determine the mean values. But none of these numbers agrees with the known parameters.

The problem of the phase difference is less intricate. The identity of period and phase of F and Φ leads easily to the conclusion that the maxima of Ĝ should follow at the distance of $\frac{1}{2}$ synodic month the minima of Φ[13]. In fact, however, this phase difference is not $\frac{1}{2}$ but $\frac{1}{2}(1-\varepsilon)$. As can easily be seen by considering the "true function", which represents the lunar velocity, this arrangement has the following consequence: there exists a certain Saros interval such that the moon is at its maximum velocity $\frac{1}{2}(1-\varepsilon)$ mean synodic months before the mean conjunction at the beginning of the Saros and $\frac{1}{2}(1+\varepsilon)$ mean synodic months before the last mean conjunction of the Saros. I cannot say, however, whether this kind of symmetrization is significant or not.

The problem of the function Φ will not be solved before we understand the numerical relations between the mean values of Φ, F, and Ĝ. It is now

[13] Cf. ACT I p. 59 Fig. 25.

clear that the two previous attempts to explain the nature of column Φ can be discarded: Kugler's hypothesis that Φ represents the apparent lunar diameter and my own suggestion to connect Φ with the elongation.[13a] It is now certain that Φ measures time in comparing the length of lunations one Saros apart. It is on this basis that the solution of the remaining questions has to be sought.

Transcription.

Obv. Beginning destroyed.

Section 1.

1. [. . . . .]. . . . .[. . . . .
2. [. . . . .] E-*ma* 14,48,53,20 mu-meš [. . . . . .
3. *šá* 18 mu-meš *šá* E-ma 13,39,35,36 [. . . .

Section 2.

4. 17,46,40 a-rá 9,20 DU-*ma* 2,45,55,[33,20 tab *u* lal]
5. ta *muḫ-ḫi* zi *sin* gal en *muḫ-ḫi* zi *sin* tur . . [. . .
6. 1,22,39,49,30 *si-man šu-ú ina* 18 mu-meš . . [. . . .
7. *sin a-n*[*a*] *šamáš* kur *ù ina* 18 mu-meš . . [. . . .
8. ta 3,13,21,4 danna nim-*ma re-ḫi* [. . . . .
9. 1,5,4,22,30 DU-*ma lu* 1,22,39,[49,30 1,16,]13,[10,11,24,36]
10. a-rá 1,5,4,22,30 DU-*ma* 1,22,39,49,30 . . .13 [. . . . .
11. ta *muḫ-ḫi* zi gal en *muḫ-ḫi* zi tur 1,16,13,10,11,2[4,36]
12. a-rá 9,20 DU-*ma* 11,51,22,55,6,29,36 a-rá 10 [DU-*ma*]
13. 1,58,{39,}33,49,11,4,56 27,33,16,30 blank [. . . . . . .]
14. *ù* 1,58,33,49,11,4,56 *a-na muḫ-ḫi a-ḫa-meš* gar-gar[-*ma*]
15. 29,31,50,19,11,4,56 šit áb kur *ina* múrub áb *aná* áb [. . .]
16. *ina* 29 me *ina* 3,11,1,6,29,36 *si-man sin a-na šamáš* kur *si-man šá a-na tar-ṣi* [. . .]
17. 1,53,56,47,24,2,40 *si-man šá* . . . *a-na tar-ṣi* 1,58,3[3,49,11,4,56]
18. 4,28,5,25,45,9,30 2,45,55,33,20 *šá* áb a-rá 9,20 DU-*m*[*a*]
19. 25,48,38,31,6,40 tab *u* lal *šá si-man-meš* 2,45,55,33,20 [. . .]
20. a-rá 7 DU-*ma* 19,21,28,53,20 19,16,51,6,40 ta *lìb* nim-*m*[*a*]
21. *re-ḫi* 4,37,46,40 $\frac{1}{2}$-*šú* GIŠ-*ma* 2,18,53,20 2,18,53,20
22. ta 2,45,55,33,20 ta *lìb* nim-*ma re-ḫi* 2,43,{42,43,}36,40

[13a] Kugler, Mondrechnung p. 125 ff. and ACT p. 44 f. respectively.

23. a-rá 9,20 DU-*ma* 25,27,2,13,20 blank 25,27,2,13,20
24. *ù* 51,37,17,2,13,20 gar-gar-*ma* 1,17,4,19,[1]5,33,20
25. ki . . . 3,11,1,6,29,26 blank *šá s*[*i-man*]

Section 3.

26. ta zi gal en zi tur 17,46,40 a-rá 12,38,45 DU-*ma* zi [gal]
27. 17,46,40 a-rá 50 DU-*ma* ta *muḫ-ḫi* zi *sin* gal en *muḫ-ḫi* z[i *sin* tur]
28. ta tur en gal 17,46,40 a-rá 2,25,37,30 DU-*ma* zi tur UL-LA-NI

Section 4.

29. [*ina* . . . . m]e 3,1,32 k[i D]U *ù*? zi-*šú* BE ta zi-*šú* gal
30. [. . . . .]4,55,46,40 me 1,.,46,1,30 DU
31. . . . .]30 DU *ina* 5,6,9,33,20 me
32. [. . . . .?4]8,45 me 3,1,32 ki DU
33. [. . . . .]. . .[. . . . m]e? zi *ina* DI zi gal? 1?-en? . . .[. . . .]
34. [. . . . .]. . . . . .[. . . . . .

Reverse. Probably little missing between obv. and rev., if anything.

Section 5.

1. [. . . . .]30? 1? 16,20 zi? 9,6,26 . . [. . . . .

Section 6.

2. [. . . . . 1,16,13,10,]11,24,36 zi *ina* ki zi [. . . .]
3. [. . . . . .z]i gal $\frac{1}{2}$? 3,30,?48,29,30,30 zi
4. [. . . . . .1]8 mu-meš *si-man šá* gal 2,13,20

Section 7.

5. [. . .]. . .[. . . . .]8 blank 1[0. . . . . .]

Section 8.

6. 23,52,13,20 *šá* 18-mu-[meš. . . . . .1]4,4,26,40 blank 7 áb

Section 9.

7. 34,24,51,21,28,53,2[0 . . . . . .*š*]*á si-man*-meš *šá* áb *aná* áb
8. 23,33,56,2,57,4[6,40 . . . .*š*]*á* ki *si-man* tab

Section 10.

9. 4,53,47,29,40,44,26,40 *s*[*i-man*. . . . .?]18,16,5,19,15,33,20 *si-man*

Section 11.

10. [1]3,46,38,15 me ta zi gal en z[i tur] a-rá 6 DU-*ma*
11. 1,22,39,49,30 danna me *a-na* 1,5,4,22,30 šeš-meš BAR-*ma*
12. 1,16,13,10,11,24,36 blank *šá*? u[š-m]eš

Section 12.

13. 17,46,40 tab *u* lal *šá* 18 mu-meš [a-rá] 1,5,4,22,3[0 DU-*ma* 19,]16,51,6,40
14. $\frac{1}{2}$-*šú* GIŠ-*ma* ki 1,16,13,10,1[1],2[4,]36 tab-*ma* [. . . . .]4,36
15. *aná tar-ṣi* 2,13,20 zi gal 1,4,5[4?,. . . . .] zi tur

Section 13. Except for a few traces, only beginnings of lines are preserved.

16. 17,46,40 tab *u* lal *šá* 18 mu-[meš . . .
17. 2,45,55,33,20 [. . .
18. 25,48,38,31,6,[40 . . . .
19. [. u]š *sin ina* ki zi [. .]. . z[i . . . .
20. [. . .]1,30,3?6 [. . . . 21. [. . .]-*ú* . . [. . . .
22. ta .[. . . . 23. . . .[. . . . 24. *si-man* [. . . .

Section 14.

25. *ki-i* [. . . . .] zi tur
26. 1,6,41,30[. . . . . z]i gal
27. 1,25,42,4[0 . . .]44[. . . . .]. . [. .] blank
28. 1,6,42[. . . . . . .]48,4,30[. . . . . .z]i gal 1,5,4,22,30
29. $\frac{1}{2}$-*šú* GIŠ-*ma* a-rá 17,46,40 DU-*ma* 9,22,33,13,4[0] tab *u* [lal]
30. 1,25,58,17,38,40 *a-na tar-ṣi* blank zi [tur]
31. 1,6,41,26,32 *a-na tar-ṣi* zi gal 19,16,51,6,40 . .[. . . . . . .]

Section 15.

32. [*a*]-*na tar-ṣi* 2,17,4,48,53,20 2,15,31,6,35,3[3,20 . . . .
33. [*a-na t*]*ar-ṣi* 2,. .,59,15,33,20 4,46,42,57,46,[40 . . . . .
34. [*a-na tar-ṣi* 1,57,47,]57,46,40 5,15,2[8,23,37,46,40 . . . . .

Section 16.

35. [. . . . .] blank [. . . . .
36. [. . . . .]. . . 57[. . . . . .

Rest destroyed.

Critical Apparatus.

Obverse.

5. End of line: two vertical wedges visible, thus 2 or 3 possible.
6. End of line: 10 or *ù*. . . . . ?
7. End of line: traces of numbers which could be 11,4,3,6,2[0 . . . or *ù* 3,6,2[0 . . . .
8. 3,13,21,4: also 15 or 16 instead of 13 not excluded.
10. End of line: 13 or 23 seems certain. The preceding traces could be 30 or 20.
11. Last number: the 11 is badly written, perhaps a correction of a 9.
13. {39}: superfluous digit without influence on computation. The correct number appears in the next line.
16. *ina* 29 me: written on the left edge [Sachs].
17. *šá* . . . : the *šá* is followed by an erasure, probably of a second *šá*.
18. . . . .,9,30 2,45,. . . : there is no space between 30 and 2, and one would read . . .9,32,45,. . . were it not for the context. Furthermore, the last digit of the first number should be 20, not 30.
22. {42,43}: superfluous digits; scribal error without influence on subsequent numbers.
25. ki: followed by an erasure or a destroyed sign.
    . . .,29,26: sic, instead of . . .,29,36 (cf. line 16).
28. 2,25,37,30: also 38 or 36 would be possible instead of 37.
33. DI: probably error for KI.

Reverse:

1. Could be the last line of Section 4.
3. $\frac{1}{2}$: or *ina* 1 ?
   3,30,48,. . . : the 40 is very uncertain.
8. 23,33,. . . : reading of 20 very uncertain.
9. . .]18: or more tens before 8.
   . . .19. . . : could also be read *ina* 9.
12. *šá*: or *aná*?

14. [. . . . .]4,36: the available space fits very well the expected length of 5 digits.
28. . .]48,4,30: or]58,4,30.
33. 2, . ,59,15 . . : reading very uncertain since the lower part of the numbers preceding 15 is broken away.
36.] . . . 57[: or . .]22,58[. . .

Translation and Commentary.

Section 1. The beginning of the text is so badly damaged that only very little can be said about its contents. The number in line 2 can be explained as

$$14,48,53,20 = 50 \cdot 17,46,40 = 50 \cdot \varphi.$$

The same factors appear also in Section 3; the present passage seems to indicate that "years" are meant, though this seems very unlikely in view of the fact that 17,46,40 is only a little more than one hour (cf. note 7).

In line 3 the 18-year cycle is mentioned. The subsequent number occurs nowhere else; the element E in the term E-*ma* is probably a form of *qabū* "speak, declare".

Section 2. This section covers the major part of the obverse. At the beginning we read:

4. 17,46,40 multiply by 9,20 and 2,45,55,[33,20 add and subtract.]
5. From high lunar velocity to low lunar velocity . . [. . . . .]
6. 1,22,39,49,30 (is) its duration; in 18 years . . [. . . . .]
7. the moon reaches the sun and in 18 years . . [. . . . .]
8. subtract from 3,13,21,4 danna and the remainder [. . . . .]

Line 4 contains the statement that

$$0;0,17,46,40^{\mathrm{H}} \cdot 9;20 = 0;2,45,55,33,20^{\mathrm{H}}$$

or

$$\varphi \cdot \frac{1}{\varepsilon} = d_{\Phi}. \tag{1}$$

Lines 5 and 6 introduce a new constant, namely,

$$c = 1,22;39,49,30^{\mathrm{H}} \tag{11}$$

the duration of half the anomalistic month—"from high to low lunar velocity" as the text puts it. Since, according to Section 11, (11 a) $$1,22;39,49,30^{\mathrm{H}} = 6 \cdot 13;46,38,15$$

we see that the length of the anomalistic month corresponding to (10) is

(11 b) $$2 \cdot 13;46,38,15^{\mathrm{d}} = 27;33,16,30^{\mathrm{d}}$$

a value not attested in ACT-texts.[14]

I do not dare restore the numbers connected with the 18-year cycle (lines 6 and 7) nor do I know the meaning of 3,13,21,4 danna (= 6,26,42,8°?) in line 8.

There follows a derivation of the length of the synodic month from the anomalistic month:

9. [What] should be multiplied by 1,5,4,22,30 so that 1,22,39,[49,30 (is the result)? 1,16,]13,[10,11,24,36]
10. times 1,5,4,22,30 is 1,22,39,49,30 ....[..... (which is the time)]
11. from high lunar velocity to low lunar velocity. 1,16,13,10,11,2[4,36]
12. multiply by 9,20 and (the result) 11,51,22,55,6,29,36 multiply by 10 [and]
13. 1,58,{39,} 33,49,11,4,56 (is the result). 27,33,16,30
14. and 1,58,33,49,11,4,56 add together and
15. 29,31,50,19,11,4,56 is the number (of days) of the month.

On the basis of Sections 11 and 12 we can explain the first number in line 9 as[15]

$$1,5;4,22,30 = \frac{19,16,51,6,40}{17,46,40,0} = \frac{\varDelta_{\Phi}}{\varepsilon \cdot d_{\Phi}} = \frac{1}{2\,\varepsilon} P_{\Phi}.$$

The division which follows means[16]

$$\frac{1,22;39,49,30^{\mathrm{H}}}{1,5;4,22,30} = 1;16,13,10,11,24,36^{\mathrm{H}} = \frac{\frac{1}{2} m_{\mathrm{a}}^{\mathrm{H}}}{\frac{1}{2\,\varepsilon} P_{\Phi}} = \varepsilon \frac{m_{\mathrm{a}}^{\mathrm{H}}}{P_{\Phi}}$$

where $m_{\mathrm{a}}^{\mathrm{H}}$ represents the length of the anomalistic month, measured in large-hours according to (11). If $m_{\mathrm{a}}^{\mathrm{d}}$ denotes the corresponding value expressed in days, as in (11b), we have for the next step

[14] This is not very significant, however, since only approximate values for the anomalistic month are known from ACT (I p. 59: 27;33,1,... p. 76: 27,33,20). A very close value results from the data in the Surya Siddhanta I, 37–39: 27;33,16,32,...

[15] For $\varDelta\Phi$ = 19,16,51,6,40 cf. ACT I p. 44; for the denominator cf. the equations (1) and (3) above.

[16] The result is not quite accurate since the two last digits should be 25,46... instead of 24,36. The restoration in lines 9 and 11 is nevertheless certain on the basis of the product given in line 12 as well as from Section 11.

$$1;16,13,10,11,24,36 \cdot 9;20 \cdot 0;10 = 1;58,33,49,11,4,56^{d}$$
$$= \varepsilon \frac{m_a^H}{P_\Phi} \cdot \frac{1}{\varepsilon} \cdot \frac{1}{6} = \frac{m_a^d}{P_\Phi}.$$

Since the periods of F and Φ are identical we can write for the last step

$$27;33,16,30 + 1;58,33,49,11,4,56 = 29;31,50,19,11,4,56^{d}$$
$$= m_a^d + \frac{m_a^d}{P_F} = m_a^d \left(1 + \frac{1}{P_F}\right) = m_a^d\, p_F ,$$

where

$$p_F = \frac{P_F + 1}{P_F}$$

is the period, measured in terms of the mean synodic month, of the zigzag function which represents the lunar velocity. Thus

$$m_a^d \cdot p_F = m_s^d$$

is the length of the mean synodic month expressed in days. As in the case of the anomalistic month the value $29;31,50,19,11,4,56^{d}$ is slightly different from the values attested in ACT.[17]

Lines 15 to 18 swarm with difficulties of syntax as well as of contents, probably increased by scribal errors. What kur "reach" in line 15 refers to, I do not know. Then I consider as one sentence the second half of line 15 and the first half of the next line:

> For mean velocity month by month in 29 days (and) in the time of 3,11,1,6,29,36 the moon reaches the sun.

In the preceding passages we had reached the result that the mean synodic month had the length of $29;31,50,19,11,4,56^{d}$. If we ignore, as is usual for column G, the 29 days and convert the remaining fractions into large-hours we obtain:

$$0;31,50,19,11,4,56 \cdot 6 = 3;11,1,55,6,29,36^{H}$$

for the excess, that is, the number quoted in the text with the omission of the 55 in the fourth place. I can see no explanation other than the assumption of a scribal omission. Since all that follows is based on the incorrect figure, the remaining operations in this section cannot be taken as authoritative theory.

[17] The standard value is $29;31,50,8,20^{d}$; cf. ACT I p. 70.

The next sentences (lines 17,18) seem to say

the duration which is opposite 1,53,56,47,24,2,40
the duration which is opposite 1,58,3[3,49,11,4,56]
(is?) 4,28,5,25,45,9,20.[18]

The terminology "(*a*) is opposite (*b*)" is common in the procedure texts which associate values of G with given values of Φ.[19] In numerical respects, we find no agreement with the rules found in ACT. The value 1,53,56,47, 24,2,40 is smaller than the minimum 1,57,47,... of Φ, and with Φ = 1,58,33,49,11,4,56 would belong either the value G = 4,56,0,0,0,0,0 (for increasing Φ) or G = 4,21,26,7,40,37,20 (for decreasing Φ), neither showing any relation to the 4,28,5,... of the text. The latter number is the total of numbers in lines 24 and 25, though influenced by the error in line 16.

The final part of Section 2 is again in disagreement with the scheme familiar from ACT but it at least shows arithmetical consistency.

18. 2,45,55,33,20 of the month multiply by 9,20 and
19. 25,48,38,31,6,40 (is the) increase and decrease of the duration. 2,45,55,33,20
20. multiply by 7 and 19,21,28,53,20 (is the result). Subtract from it 19,16,51,6,40 and
21. the remainder is 4,37,46,40. Halve it and 2,18,53,20 (is the result). 2,18,53,20
22. subtract from 2,45,55,33,20 and the remainder is 2,43,36,40[20]
23. Multiply (it) by 9,20 and 25,27,2,13,20 (is the result). 25,27,2,13,20
24. and 51,37,17,2,13,20 add and 1,17,4,19,15,33,20 (is the result);
25. with 3,11,1,6,29,36[21] of the duration (it gives a total of 4,28,5,25,45,9,20).

The meaning of these operations can be described as follows. First we are reminded that

$$0;2,45,55,33,20^{\mathrm{H}} \cdot 9;20 = 0;25,48,38,31,6,40^{\mathrm{H}}$$

or:

$$d_{\Phi} \cdot 9;20 = d_{\hat{\mathrm{G}}}.$$

[18] The text has 30 for the last digit but 20 is required by the subsequent computations.
[19] Cf., e. g., ACT I p. 204, p. 253, etc.
[20] The text gives here incorrectly 2,43,*42*,*43*,36,40 but all subsequent numbers are correct.
[21] The text gives for the last digit 26 instead of 36; the correct value is found in line 16.

Then we turn to the function Φ and form

$$\begin{array}{rcll} 0;2,45,55,33,20^{\mathrm{H}} \cdot 7 & = & 0;19,21,28,53,20^{\mathrm{H}} & = 7 \cdot d_\Phi \\ & - & 0;19,16,51,6,40^{\mathrm{H}} & = -\varDelta_\Phi \\ \text{thus:} & & 0;0,4,37,46,40^{\mathrm{H}} & = 7\ d_\Phi - \varDelta_\Phi \end{array}$$

and

$$\frac{1}{2} \cdot 0;0,4,37,46,40^{\mathrm{H}} = 0;0,2,18,53,20^{\mathrm{H}} = \frac{1}{2}(7\ d_\Phi - \varDelta_\Phi)$$

and thus

$$0;2,45,55,33,20^{\mathrm{H}} - 0;0,2,18,53,20 = 0;2,43,36,40 = d_\Phi - \frac{1}{2}(7\ d_\Phi - \varDelta_\Phi).$$

This result is now transformed into values of $\hat{\mathrm{G}}$ by multiplying it by 9;20

$$0;2,43,36,40^{\mathrm{H}} \cdot 9;20 = 0;25,27,2,13,20^{\mathrm{H}} = d_{\hat{\mathrm{G}}} - \frac{1}{2}(7\ d_{\hat{\mathrm{G}}} - \varDelta_{\hat{\mathrm{G}}})$$

to which is added

$$0;51,37,17,2,13,20^{\mathrm{H}} = 2\,d_{\hat{\mathrm{G}}}$$

with the result

$$1;17,4,19,15,33,20^{\mathrm{H}} = \frac{1}{2}(\varDelta_{\hat{\mathrm{G}}} - d_{\hat{\mathrm{G}}}).$$

Finally mention is made of the number

$$\mu = 3;11,1,6,29,36^{\mathrm{H}}$$

which is known from line 16 as the length of a synodic month (beyond 29 days) which corresponds to a mean anomalistic month and can therefore be considered as the real value of the mean synodic month, though influenced by an omission of 55 in the fourth place.

Section 2 ends without giving the result of the addition which is indicated by the particle ki "with". If we add the two last mentioned figures we find

$$1;17,4,19,15,33,20^{\mathrm{H}} + 3;11,1,6,29,36^{\mathrm{H}} = 4;28,5,25,45,9,20^{\mathrm{H}}$$

$$\frac{1}{2}(\varDelta_{\hat{\mathrm{G}}} - d_{\hat{\mathrm{G}}}) + \mu = M - \frac{1}{2}\,d_{\hat{\mathrm{G}}}\,.$$

We have already encountered the same number at the end of the preceding section (line 18) as "the duration which is opposite 1;58,3[3,49,11,4,56]". Assuming the correctness of my restoration of the last number it represents the difference between the mean synodic and anomalistic month (cf. lines 13 and 14).

An explanation of these operations can, to a certain extent, be given as follows. Since the period of Φ is slightly less than 14 mean synodic months[22] $7\,d_{\Phi}$ slightly exceeds the amplitude Δ_{Φ} and $\frac{1}{2}(7\,d_{\Phi}-\Delta_{\Phi})$ indicates by how much an interval of $7\,d_{\Phi}$ width exceeds in a symmetric position the extrema of Φ (cf. Fig. 2). Consequently $d_{\Phi}-\frac{1}{2}(7\,d_{\Phi}-\Delta_{\Phi})$ indicates the distance of the first and last interior point of a branch from its nearest extremum. And since multiplication by 9;20 transforms all numbers into the corresponding quantities of Ĝ, we see that $0;25,27,2,13,20^{H}$ is the amount which separates the extrema of Ĝ from the nearest values in an arrangement as represented in Fig. 2.

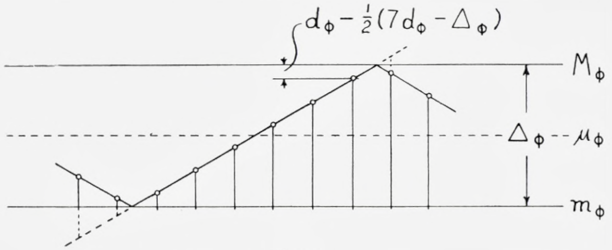


Fig. 2.

If we consider $\mu = 3;11,1,6,29,36^{H}$ as the mean value of Ĝ then

$$\mu+\frac{1}{2}(\Delta_{\hat{G}}-d_{\hat{G}}) = M-\frac{1}{2}d_{\hat{G}} = 4;28,5,25,45,9,20^{H}$$

is the value of Ĝ one half interval before its maximum. The motivation for this probably lies in the fact that it is this value which would, in general, be associated with the minimum of Φ or F.[23]

Section 3.

26. From high velocity to low velocity: 17,46,40 multiply by 12,38,45 and the [high] velocity (is the result).
27. 17,46,40 multiply by 50 and from the high lunar velocity to the [low lunar] velocity (is the result).

[22] $P\Phi = \frac{1,44,7}{4,28} = 13;56,39,\ldots$; cf. above p. 5 (5).

[23] Cf. ACT I p. 32 and p. 59.

28. From low to high: 17,46,40 multiply by 2,25,37,30 and the low (velocity ....(is the result).

I do not understand the meaning of these operations. In all three cases $1/\varepsilon$ is multiplied by a facto rwithout, however, giving the result. One would find

3,44,50,53,20 14,48,53,20 48,8,53,20

respectively. The second number occurs also in the damaged second line of Section 1.

Section 4 is badly damaged. So far as I can make out, certain intervals of days are listed together with the corresponding lunar motion. Unfortunately not a single pair is completely preserved and only one number is denoted as ki "degrees", thus presumably integer degrees. The remaining numbers in all probability contain fractional parts. I interpret the contents as follows:

[in ......] days 3,1,32° travel
and (?) its velocity; if (?) from its high velocity [........]
[in ....]4,55,46,40 days 1,0,46,1,30 travel
[in ..... days ....] 30 travel
in 5,6,9,33,20 days [......travel]
[in .....4]8,45 days 3,1,32° travel.

Since we are dealing with relatively large numbers of days, the results should not be much different from a motion with mean velocity. This would lead in the first line and in the last to an interval of about 13,46 days. Neither this nor any of the other numbers quoted is otherwise known.

Sections 5 to 10. Not much more can be done with the beginning of the reverse than to identify some of the numbers or words.

Line 1 concerns a velocity.

Line 2 I have tentatively restored from obv. 11 and rev. 12 and 14[24] though the association with velocities might make it suspect. Line 4 refers to the 18-year period and quotes the number 2;13,20 which, for decreasing Φ, marks the beginning of increasing G above its minimum value[25] of $2;40^{H}$. In certain passages of procedure texts the same number 2,13,20 is used as

[24] Cf. p. 13.
[25] Cf. ACT I p. 60 Fig. 26 and table.

if it were a technical term.[26] In our text it occurs again in Section 12 (line 15).

None of the large numbers in lines 6 to 9 is otherwise known. The repeated references to *si-man* "duration" points to a connection with column G but even an interpretation as fractions of days does not lead to familiar numbers.

Section 11.

10. 13;46,38,15 days from high velocity to [low] velocity. Multiply (it) by 6 and
11. 1,22;39,49,30 .... (is the result). Divide (it) into 1,5;4,22,30 parts and 1;16,13,10,11,24,36 (is the result) ...

The use of the units danna (= 30°) in line 11 must be a mistake for uš-meš (degrees) which are mentioned at the end of line 12. What me in line 11 means is not clear.

The purpose of this operation has already been explained in Section 2. The first two large numbers give the half of the length of an anomalistic month, counted in days and large-hours respectively. For the subsequent division see above p. 13 and the next section.

Section 12.

13. 17,46,40, the difference for 18 years, [multiply by] 1,5;4,22,30 [and (the result is) 19,]16,51,6,40.
14. Halve it and add to 1;16,13,10,11,24,36 and [............]4,36 (is the result).
15. Opposite 2,13,20, the high velocity, 1,4,5[4(?) ....] low velocity.

Again using the notation of Section 2 we see that line 13 computes

$$\varepsilon \cdot d_{\Phi} \cdot 1{,}5;4{,}22{,}30 = \Delta_{\Phi} = 0;19{,}16{,}51{,}6{,}40^{\mathrm{H}}.$$

The operation in the next line is most naturally interpreted as

$$\frac{1}{2}\Delta_{\Phi} + 1;16{,}13{,}10{,}11{,}24{,}36 = \frac{1}{2}\Delta_{\Phi} + \varepsilon\frac{m_a}{P_{\Phi}}$$
$$= [1;25{,}51{,}35{,}44{,}4]4{,}36^{\mathrm{H}}.$$

It would be less plausible to refer the "it" to 17,46,40 and thus restore

[26] Cf. ACT I p. 212.

$$\frac{1}{2}\varepsilon d_{\Phi}+\varepsilon\frac{m_a}{P_{\Phi}}=0;0,8,53,20+1;16,13,10,11,24,36$$
$$=[1;16,22,3,31,2]4,36^{\mathrm{H}}.$$

Unfortunately neither of these combinations seems to make sense nor to explain its relation to the value 2,13,20 of Φ which is associated with the minimum of G (and thus with "high velocity" of the moon, as seems to be indicated by the text).[27]

Section 13 begins with the same sentence as Section 12. The number at the beginning of line 17 is d_{Φ}. Line 18 begins with $d_{\hat{G}}$, which can be obtained from d_{Φ} by multiplication by 9;20 (cf. obv. 18 and 19).

Sections 14 to 16. Nothing can be connected before line 28/29, which seems to indicate that

$$\frac{1}{2}\cdot 1,5;4,22,30\cdot 17,46,40=9,22,33,13,40.$$

This result, however, is incorrect since we know from Section 12 that its value should be

$$\frac{1}{2}\Delta_{\Phi}=9,38,25,33,20.$$

The result of the text is furthermore described as "add and [subtract]", which is the standard terminology for the difference of a zigzag function (e.g., obv. 19) but would not apply to $\frac{1}{2}\Delta_{\Phi}$.

The next two lines, 30 and 31, are numerically correct in so far as

$$1;25,58,17,38,40^{\mathrm{H}}=1;6,41,26,32^{\mathrm{H}}+0;19,16,51,6,40^{\mathrm{H}}$$

where the last-mentioned number is the value of Δ_{Φ}. The first number is associated with low velocity, the second with high velocity. Since their difference is Δ_{Φ} one would expect to find here the maximum and minimum of Φ respectively. In fact, however, these values are

$$M_{\Phi}=2;17,4,48,53,20 \quad \text{and} \quad m_{\Phi}=1;57,47,57,46,40$$

and are in no obvious relation to the above numbers.

[27] Cf. also the end of Section 6 and our commentary to it (p. 9 and 19).

The numbers M_{Φ} and m_{Φ} do occur, however, in the next section (lines 32 and 34 respectively) where they are associated with two numbers whose difference is $\Delta_{\hat{G}}$ but in the opposite order of numerical value in accordance with the general rule for the phases of Φ and $\hat{G}$; but the numbers themselves

$$2,15,31,6,35,3[3,20] \quad \text{and} \quad 5,15,2[8,23,37,46,40]$$

are again different from the extrema of $\hat{G}$

$$m_{\hat{G}} = 2;4,59,45,11,6,40^{H} \quad \text{and} \quad M_{\hat{G}} = 5;4,57,2,13,20^{H}$$

though the discrepancy is no longer so great as with Φ in the preceding section.

In line 33 we find a pair of numbers which agrees with the scheme known from ACT (p. 60):

$$\Phi = 2;0,59,15,33,20^{H}(\uparrow) \text{ corresponds to } G = 4;46,42,57,46,40^{H}.$$

This pair is of particular importance since the value of G is the greatest value for which G and $\hat{G}$ are identical. Since this pair agrees with the known scheme it seems tempting to consider the values which are associated with M_{Φ} (line 32) and with m_{Φ} (line 34) as errors:

$$2;15,31,6,35,3[3,20 \quad \text{should be} \quad 2;16,31,6,40$$

and

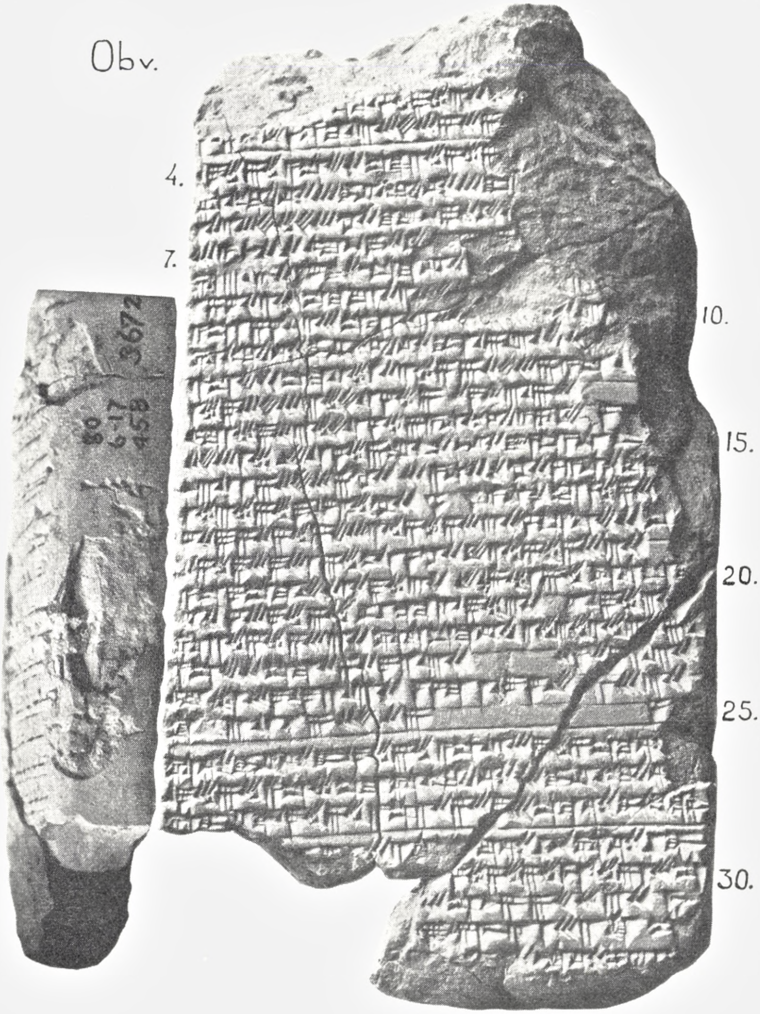
$$5;15,2[8,23,37,46,40 \quad \text{should be} \quad 5;16,28,23,42,13,20$$

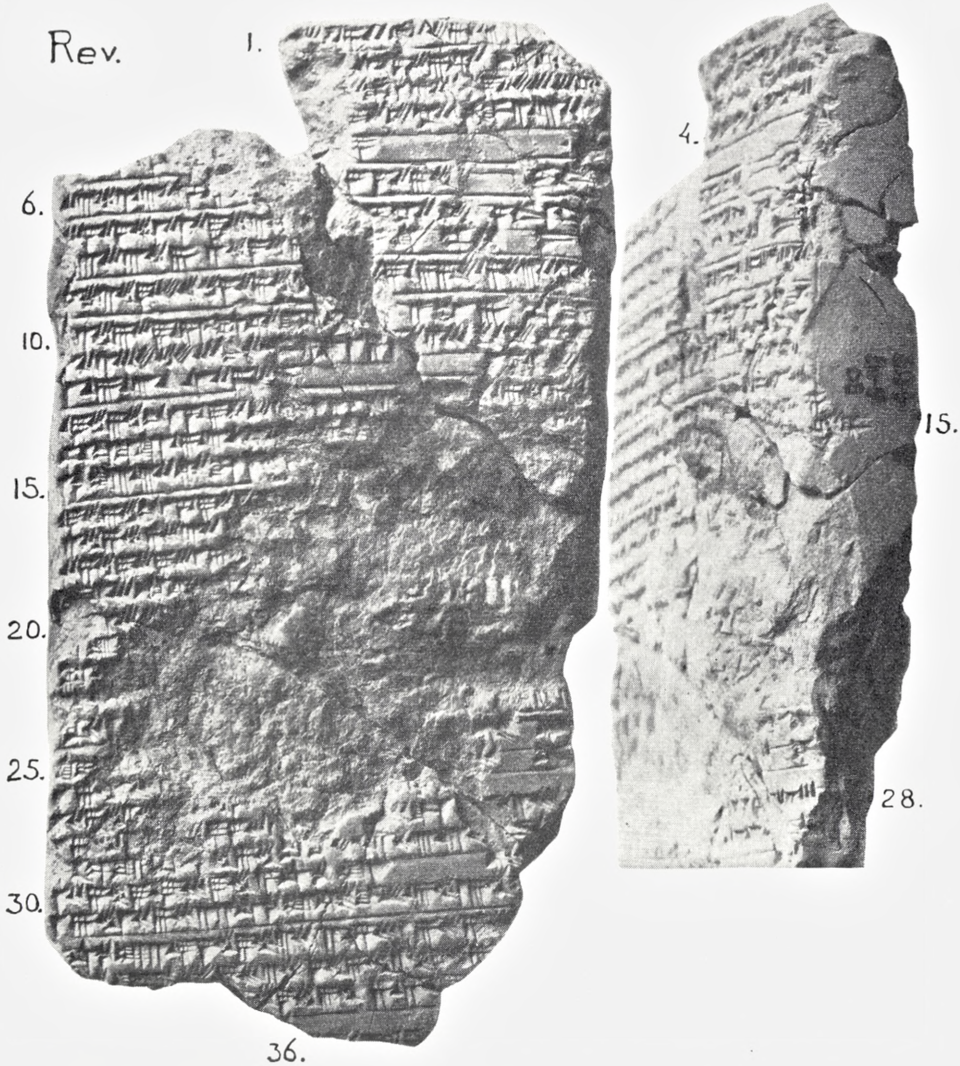
respectively. In this case Section 15 would have listed three values of $\hat{G}$ namely the values which correspond to the extrema of Φ and the maximum value for which $G = \hat{G}$.

Indleveret til selskabet den 15. januar 1957.
Færdig fra trykkeriet den 8. maj 1957.

Plate I

Obv.

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